Stochastic Optimal Timber Harvest Problem and the Value of Carbon Sequestration: A Real Options Analysis

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Outline

Price Risk in Forest Management

- Traditional net present value methods ignore risk
- Stochastic price fluctuations pose significant risk to forest owners
- Ignoring risk leads to inaccurate valuation and sub-optimal management
- Real options provide a practical approach to risk analysis and management
- Real option analysis of the stochastic optimal timber harvest problem will be presented

Options

Basic Definitions

Real American Call Option

- An `option` is a right to buy/sell an asset at a fixed price
- `Call` property implies the right to buy
- `American` property allows exercising any time
- In forestry, timber is a `real` (physical) asset

Basic Premise

- Timber ownership is the right to buy timber for harvest cost and sell it on the open market at prevailing price
- This formulation leads to a real American call option on the value of timber

Problem Formulation

Real options are alterations of the basic stochastic optimal stopping problem

$$\pi(S_0) = \sup_{\tau^* \in \mathbb{R}^+} \mathbb{E}[d_0^{\tau^*} Q_{\tau^*} (S_{\tau^*} - C)^+ | S_0] \quad (1)$$

- $\pi(S_0)$ = expected discounted value of optimal harvest
- S_0 = starting timber price
- τ^* = optimal harvest time
- $d_0^{\tau^*}$ = discount factor
- Q_t = yield function
- S_{τ^*} = stochastic price of timber per unit volume
- C = harvest cost per unit volume

American Option Valuation

- Early exercise possibility (finding τ^*) greatly complicates American option valuation
- Closed-form solutions of American option valuation problems limited to cases inapplicable in forestry
- Valuation of American options is an area of active research
- Solutions approximated by the use of numerical methods:
  - `Finite Differences`: Fast, moderately flexible, difficult
  - `Trees`: Fast, easy to implement, not very flexible
  - `Monte Carlo Methods`: Very flexible, easy, not very fast
- A Monte Carlo algorithm was applied to the stochastic optimal harvest problem

Stochastic Price Model

- Many models for price behavior are available
- Choice is driven by available historical data and economic theory
- Logarithmic mean-reverting process was used to model timber and CO_2 prices

$$dS_t = \kappa(\mu - \ln S_t)\, S_t\, dt + \sigma S_t\, d\mathbb{W}_t \tag{2}$$

- S_t = asset price
- κ = rate of mean reversion
- μ = log of long term price
- σ = price volatility
- $d\mathbb{W}_t$ is increment of the Wiener process

Stochastic Optimal Harvest Problem

Motivation

- **Properties**:
  - `Classical Faustmann Problem:` Choose rotation length to maximize bare land value over multiple harvest cycles subject to silvicultural and economic constraints
  - `Modification:` Introduce risk via stochastic prices of timber and CO_2
- **Objectives**:
  - Bare land value with stochastic timber and CO_2 prices
  - Optimal harvest strategy in stochastic settings
  - Impact of carbon sequestration on optimal harvest age

Stochastic Faustmann Problem as Real Option

IF Land ownership is viewed as the right to exchange timber for harvest cost and sell it in the market at prevailing price

THEN Valuation of forest land under price risk parallels the valuation of a multi-period American call option

American Call	Bare Land Value
Underlying Assets	Timber and Carbon
Exercise Time	Harvest Time
Strike Price	Harvest Cost
Contract Length	Planning Horizon

Solution Algorithm

Part I

- Monte Carlo algorithm was used for its flexibility
- Based on method introduced by Ibáñez and Zapatero
- Extended to calculate value of multiple rotations
- Modified to solve problems with realistic CO_2 scenarios

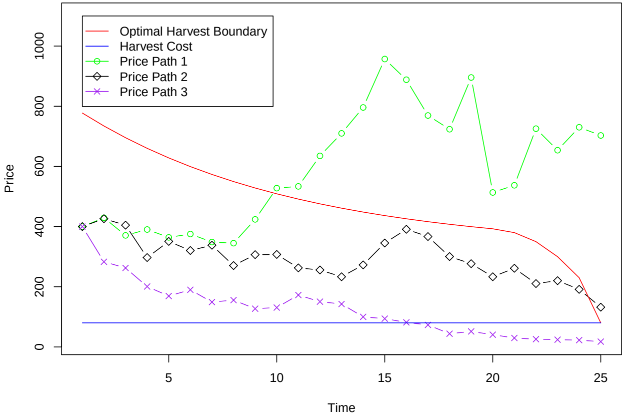
- Two-part solution:
  - Expected bare land value
  - Optimal harvest policy as function of prices and age

- Recall Equation 1:

$$\pi(S_0) = \sup_{\tau^* \in \mathbb{R}^+} \mathbb{E}[d_0^{\tau^*} Q_{\tau^*} (S_{\tau^*} - C)^+ | S_0] \quad (1)$$

Solution Algorithm

Part II



Carbon Treatment

Three basic carbon pools are considered in this study:

- **Forest Pool:** All carbon contained in a standing forest
- **Product Pool:** All carbon contained in harvested wood products
- **Substitution Pool:** All carbon not released into the atmosphere when harvested wood products displace fossil-based alternatives (Avoided emissions)

Cash Flows

- Decision at time t:

$$\pi_t = \max\left[CF_C^t + \mathbb{E}(d_t^{t+1}\,\pi_{t+1}^{NH}); CF_T^t + \mathbb{E}(d_t^{t+1}\,\pi_{t+1}^{H})\right] \quad (3)$$

- Cash flow if harvest does not occur at time t:

$$CF_C^t = \gamma\,\Delta\,Q_t\,P_C^t \quad (4)$$

- Cash flow if harvest does occur at time t:

$$CF_T^t = Q_t\left[P_T^t - \gamma\,(\alpha_F - \alpha_P - \alpha_S)\,P_C^t - C\right] \quad (5)$$

Where Q_t = yield; P_T^t = timber price; P_C^t = CO_2 price; C = harvest cost; α_F, α_P, α_S are carbon fractions in forest, product and substitution pools; and γ converts carbon in wood to atmospheric CO_2

Carbon Scenarios

- Scenarios constructed from three sets of values of α_i in Equation 5:

$$CF_T^t = Q_t \left[P_T^t - \gamma\left(\alpha_F - \alpha_P - \alpha_S\right) P_C^t - C\right] \tag{5}$$

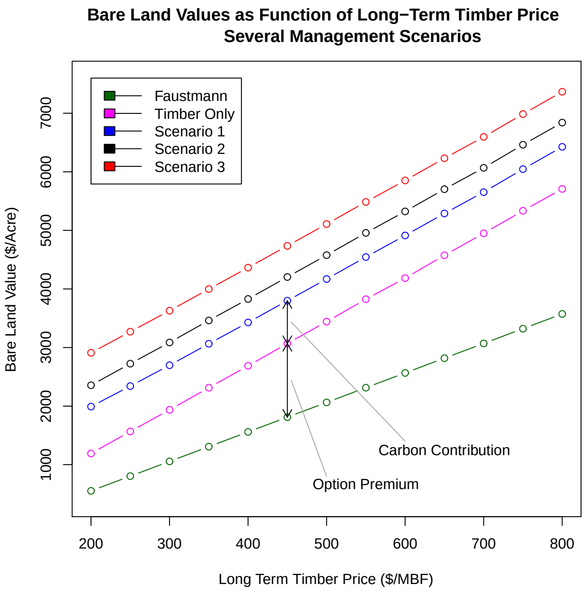
Scenario	α_F	α_P	α_S
No. 1	0.80	0.2	0.2
No. 2	0.80	0.25	1.0
No. 3	0.80	0.35	2.0

- No. 1: $\alpha_F > \alpha_P + \alpha_S \Rightarrow$ Increased harvest cost due to CO_2
- No. 2: $\alpha_F < \alpha_P + \alpha_S \Rightarrow$ Moderate CO_2 harvest revenue
- No. 3: $\alpha_F \ll \alpha_P + \alpha_S \Rightarrow$ High CO_2 harvest revenue

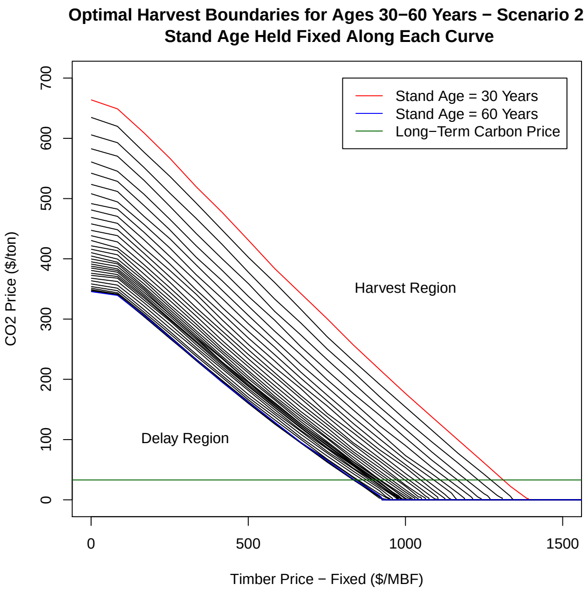
Parameter Values – Optimal Harvest Problem

Parameter	Timber	Carbon
Initial Price P^0	400 (\$/MBF)	25 (\$/ton)
Long-term Price	665 (\$/MBF)	33 (\$/ton)
Reversion Rate κ	0.33 (%/year)	4.0 (%/year)
Volatility σ	0.25 (%/year)	0.5 (%/year)
Correlation ρ	10 (%)	
Harvest Cost C	100 (\$/MBF)	
Discount Rate r	5 (%/year)	
Simulation Horizon T	100 (years)	
Harvest Time	Anytime before T (year)	
Yield Function	High-yield site in Western Washington	
Silviculture	Douglas fir regime with planting followed by a clear cut harvest	

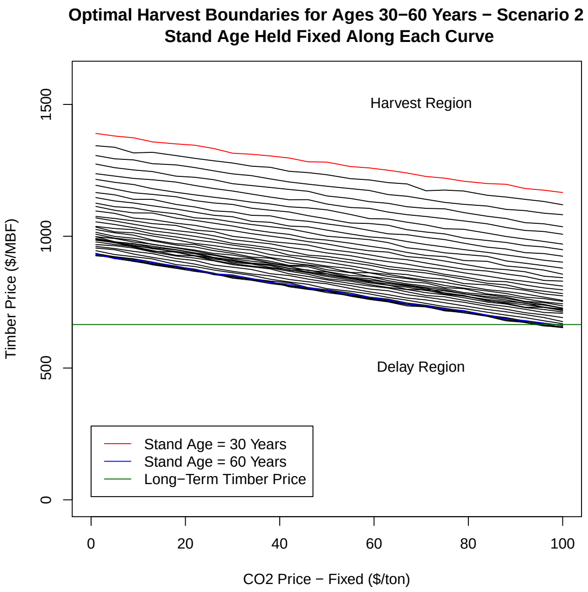
Bare Land Value & Long Term Timber Price



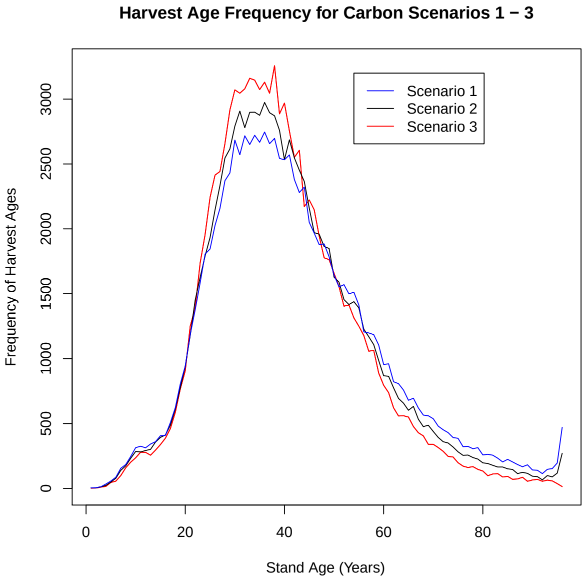
Decision Boundaries - Carbon



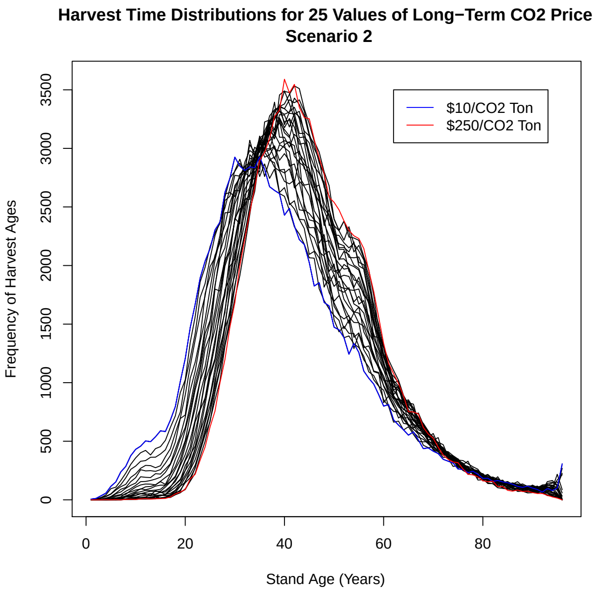
Decision Boundaries - Timber



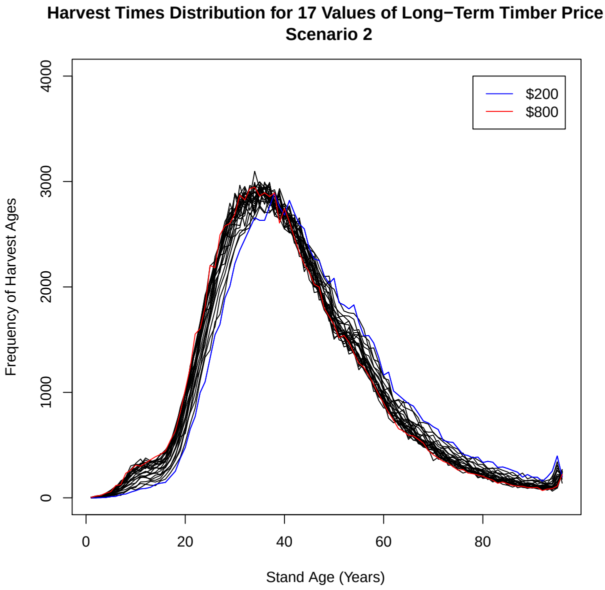
Harvest Age Frequency: Scenarios 1 - 3



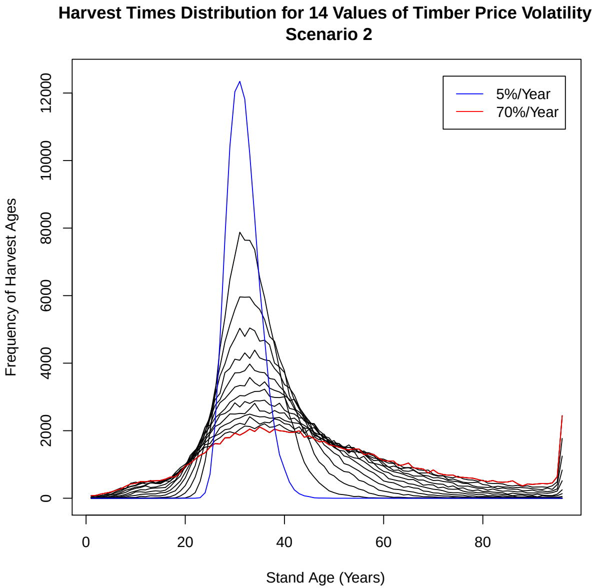
Harvest Time Frequency & CO_2 Price Sensitivity



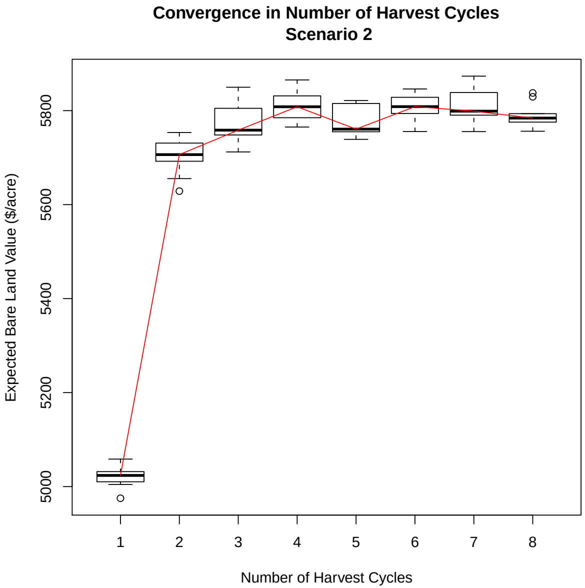
Harvest Time Frequency & Timber Price Sensitivity



Harvest Time Frequency & Timber Price Volatility



Harvest Cycle Contribution



Summary

- Stochastic price fluctuations are a significant source of risk in forest management
- Real options methodology provides a practical approach to valuation and optimal management of forestry assets in the presence of risk
- Monte Carlo is flexible and powerful framework for solving complex real options that arise in forest management

- Outlook
  - More realistic price models
  - Additional sources of risk
  - Faster, more efficient computation